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Five-Dimensional Rindler Spacetime

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We obtain solutions to the field equations for the massless scalar, massless spinor, and electromagnetic fields in a Rindler coordinate system consisting of four space dimensions and one time dimension. The solutions are shown to allow energy > momentum and so may represent massive particles even though the corresponding field equations contain no mass term. The solutions are confined to a narrow interval in the Rindler coordinate and thus, to an observer who sees only these fields, appear to exist in a three-dimensional space. We propose this description of spacetime and fields as model of the universe in which we live.

1. INTRODUCTION

The coordinates most often used in connection with a uniformly accelerated observer are Rindler coordinates (Rindler, 1966). If an observer accelerates along the z axis of a Minkowski frame such that he or she feels a constant acceleration a, then the Rindler position z and the Rindler time t of an event in the observer's frame are related to the Minkowski position z_M and the Minkowski time t_M of the event by

$$z_M = z \cosh(at) \tag{1.1}$$

$$t_M = z \sinh(at) \tag{1.2}$$

This yields the line element

$$d\tau^2 = dz_M^2 - dt_M^2 = dz^2 - a^2z^2\, dt^2 \tag{1.3}$$

The observer's Rindler position $z = L$ satisfies

$$a = \frac{1}{L} \tag{1.4}$$

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This result is obtained by using equation (1.2) and requiring that $t_M = t$ at $z = L$ when t_M and t are small, i.e., when the observer's speed in the Minkowski frame is small.

Classically, a free particle at rest in a Minkowski frame follows a trajectory in the Rindler frame given by

$$z = \frac{z_M}{\cosh(at)}$$

where z_M is constant. Classically, therefore, no free particle can remain at rest in the Rindler frame. All free particles must accelerate. For the sake of brevity, we will refer to this acceleration as a gravitational acceleration resulting from a gravitational field which exists in the Rindler frame in spite of the fact that Rindler coordinates describe a flat space-time which contains no gravitational field. This somewhat inappropriate nomenclature is justifiable because, after all, such acceleration mimics gravity very closely.

Even though, classically, free particles are required to accelerate in a Rindler frame, quantum mechanically they need not experience any such acceleration. In this paper we will obtain stationary solutions for the Klein–Gordon, Maxwell, and Dirac equations in the Rindler frame. These solutions will be seen to hover above the Rindler horizon, in spite of the ambient gravitational field. The existence of these solutions may be demonstrated by superposing a solution propagating toward the horizon with its time-reversed twin propagating away from the horizon. Such a superposition is a standing wave, stationary relative to the horizon.

While obtaining these solutions, it will suit us to work in a space-time containing four space dimensions and one time dimension, although the problem could equally well be solved in ordinary space-time (Unruh, 1976). Our reason for doing this is that it will allow us to assign a mass, if we like, to a field even in the case when no mass term is present in the field equation. This avoids the Higgs mechanism (Higgs, 1964) and ultimately yields massive fields in four-dimensional space-time. Compactification from five dimensions to four arises naturally from the presence of the gravitational field in the Rindler coordinates, which confines all fields to a thin four-dimensional slice near the horizon. This will allow us to postulate a model of the space-time in which we live as the four-dimensional remnant of a compactified five-dimensional Rindler space-time.

Throughout this paper Greek symbols will be used for indices which run from 0 through 4 and Latin symbols for indices which run from 1 through 4. The notation ∂_μ will be used to indicate the derivative with respect to x^μ and repeated indices will be summed over.

2. THE SCALAR FIELD

We define the coordinate system in which we will be working to have four space coordinates x^1, x^2, x^3, $x^4 = z$ and one time coordinate $x^0 = t$. The coordinates are Rindler coordinates with metric tensor components $g_{\mu\nu}$ given by

$$\begin{aligned} g_{00} &= -\frac{z^2}{L^2} \\ g_{11} &= g_{22} = g_{33} = g_{zz} = 1 \\ g_{ij} &= 0, \qquad i \neq j \end{aligned} \tag{2.1}$$

where L is the position of the Rindler observer.

The Klein–Gordon equation for scalar field $\phi = \phi(x^1, x^2, x^3, z, t)$ is

$$g^{\alpha\beta}(\partial_\alpha\partial_\beta\phi - \Gamma^\lambda_{\alpha\beta}\,\partial_\lambda\phi) - m^2\phi = 0 \tag{2.2}$$

where m is the mass associated with ϕ and $\Gamma^\lambda_{\alpha\beta}$ is the Christoffel symbol.

Expressing the Christoffel symbol in terms of the $g_{\mu\nu}$

$$\Gamma^\lambda_{\alpha\beta} = \tfrac{1}{2}g^{\lambda\sigma}(\partial_\beta g_{\alpha\sigma} + \partial_\alpha g_{\sigma\beta} - \partial_\sigma g_{\alpha\beta}) \tag{2.3}$$

and using equation (2.1), we find that the only relevant nonzero Christoffel symbol is

$$\Gamma^4_{00} = \frac{z}{L^2} \tag{2.4}$$

Equations (2.1), (2.2), and (2.4) yield

$$\frac{L^2}{z^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial z^2} - \frac{1}{z}\frac{\partial\phi}{\partial z} - \nabla^2\phi + m^2\phi = 0 \tag{2.5}$$

where $\nabla^2 = \partial_1{}^2 + \partial_2{}^2 + \partial_3{}^2$ is the ordinary three-dimensional Laplacian operator.

We now express the coordinate dependence of ϕ as follows:

$$\phi = f(z)\exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{2.6}$$

where $\mathbf{x} = (x^1, x^2, x^3)$, $\mathbf{p}$ is the momentum conjugate to $\mathbf{x}$, and E is an energy.

Equations (2.5) and (2.6) yield

$$(Bz)^2\frac{d^2f}{d(Bz)^2} + (Bz)\frac{df}{d(Bz)} - [(Bz)^2 + (iEL)^2]f = 0 \tag{2.7}$$

where

$$B = (p^2 + m^2)^{1/2} \tag{2.8}$$

and $p = |\mathbf{p}|$. We recognize equation (2.7) as the modified Bessel equation in coordinate Bz. Its solution is the modified Bessel function of order iEL,

$$f(z) = f(-z) = K_{iEL}(Bz) \tag{2.9}$$

Equations (2.6) and (2.9) then give us the (unnormalized) wave function ϕ,

$$\phi = K_{iEL}(B|z|) \exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{2.10}$$

We now point out an important feature of this solution. If we set $m = 0$ in equation (2.2), i.e., we remove the mass term, then the solution, equation (2.10), becomes

$$\phi = K_{iEL}(p|z|) \exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{2.11}$$

This solution exists for all E and p, and in particular for $E > p$. Equation (2.2) therefore has a solution with $E > p$ (i.e., ϕ has a mass) even when the mass term is absent. We have the possibility of introducing mass in a gauge-invariant way without introducing the Higgs field.

By multiplying equation (2.2) and its complex conjugate by ϕ^* and ϕ, respectively, subtracting them, and making use of the fact that $g^{\alpha\beta}$ is independent of x^α in the Rindler frame, we obtain

$$\partial_\mu J^\mu + \frac{1}{z} J^4 = 0 \tag{2.12}$$

where the current J^μ is given by

$$J^0 = i\frac{L^2}{z^2}(\phi^*\partial_0\phi - \phi\partial_0\phi^*) \tag{2.13}$$

$$J^k = -i(\phi^*\partial_k\phi - \phi\partial_k\phi^*), \qquad k = 1, 2, 3, 4 \tag{2.14}$$

Equation (2.12) is the statement that the covariant divergence of scalar current in the Rindler frame vanishes. [The covariant divergence of J^μ is $D_\mu J^\mu = \partial_\mu J^\mu + \Gamma^\mu_{\mu\nu} J^\nu$ and in the Rindler frame $\Gamma^\mu_{\mu\nu} = \delta^4_\nu(1/z)$. This gives $D_\mu J^\mu = \partial_\mu J^\mu + (1/z)J^4$.]

Equations (2.11) and (2.14) yield

$$J^4 = -i(K_{-iEL}\partial_4 K_{iEL} - K_{iEL}\partial_4 K_{-iEL}) = 0$$

since $K_{iEL} = K_{-iEL}$. There is no particle flux in the z direction.

For large z, we have (Arfkin, 1970)

$$K_\nu = \frac{\pi}{(2z)^{1/2}} e^{-z}\left[1 + \frac{4\nu^2 - 1}{8z} + \frac{(4\nu^2 - 1)(4\nu^2 - 9)}{2!\,(8z)^2} + \cdots\right] \tag{2.15}$$

which indicates that for $B \neq 0$, ϕ attenuates as z becomes large. Equation (2.13) then says that the probability density J^0 associated with ϕ must also attenuate. The scalar field, unrestricted as far as x^1, x^2, and x^3 are concerned, is confined in four-dimensional space to a region near $z = 0$. On a scale large compared to the extent of this region in the z direction, the field ϕ would appear to lie in a three-dimensional subspace just above $z = 0$. To an observer who exists on this scale and sees only ϕ, the space would appear to contain only three dimensions. This is the mechanism of compactification that we will employ throughout this paper. In subsequent sections we will apply it to the electromagnetic and spinor fields.

3. ELECTROMAGNETIC FIELD

To study the behavior of the electromagnetic field in five-dimensional Rindler space-time we consider the electromagnetic vector potential $\tilde{A}_\nu$. In vacuum it can be chosen to satisfy

$$g^{\lambda\mu} D_\lambda D_\mu \tilde{A}_\nu = 0 \tag{3.1}$$

where D_μ is the covariant derivative.

To simplify the examination of equation (3.1) we define the following fields:

$$V^\alpha_\mu = \frac{\partial \xi^\alpha}{\partial x^\mu} \tag{3.2}$$

$$V^\mu_\alpha = \frac{\partial x^\mu}{\partial \xi^\alpha} \tag{3.3}$$

$$A_\alpha = V^\lambda_\alpha \tilde{A}_\lambda \tag{3.4}$$

where x^μ is the Rindler coordinate and ξ^α is the Minkowski coordinate. We then have

$$D_\mu \tilde{A}_\nu = \partial_\mu \tilde{A}_\nu - \Gamma^\lambda_{\mu\nu} \tilde{A}_\lambda = V^\alpha_\nu \partial_\mu A_\alpha \tag{3.5}$$

Equation (3.1) then reads

$$g^{\lambda\mu}[\partial_\lambda (V^\alpha_\nu \, \partial_\mu A_\alpha) - \Gamma^\sigma_{\lambda\mu} V^\alpha_\nu \, \partial_\sigma A_\alpha - \Gamma^\sigma_{\lambda\nu} V^\alpha_\sigma \, \partial_\mu A_\alpha] = 0 \tag{3.6}$$

From equations (2.1) and (2.3) we find that the only relevant nonzero Christoffel symbols are

$$\Gamma^4_{00} = \frac{z}{L^2} \tag{3.7}$$

$$\Gamma^0_{40} = \Gamma^0_{04} = \frac{1}{z} \tag{3.8}$$

From equations (2.1) and (3.2) and the relation

$$g_{\mu\nu} = \eta_{\alpha\beta} V^\alpha_\mu V^\beta_\nu$$

where $\eta_{\alpha\beta} = \text{diag}(-1, 1, 1, 1, 1)$, we find

$$V^\alpha_\mu = \delta^\alpha_\mu + \left(\frac{z}{L} - 1\right)\delta^\alpha_0 \delta^0_\mu \tag{3.9}$$

Equations (2.1) and (3.6)–(3.9) yield

$$\begin{aligned} &-\frac{1}{L}\,\delta^0_\nu \partial_4 A_0 + \frac{L^2}{z^2}\,\partial^2_0 A_\nu - \nabla^2_4 A_\nu + \left(\frac{z}{L} - 1\right)\delta^0_\nu\left(\frac{L^2}{z^2}\,\partial^2_0 A_0 - \nabla^2_4 A_0\right) \\ &-\frac{1}{z}\left[\partial_4 A_\nu + \left(\frac{z}{L} - 1\right)\delta^0_\nu \partial_4 A_0\right] - \frac{1}{z}\,\delta^0_\nu \partial_0 A_4 + \frac{1}{L}\,\delta^0_\nu \partial_4 A_0 - \frac{L}{z^2}\,\delta^4_\nu \partial_0 A_0 \\ &= 0 \end{aligned} \tag{3.10}$$

where $\nabla^2_4 = \partial^2_1 + \partial^2_2 + \partial^2_3 + \partial^2_4$.

We now seek a solution to equation (3.10) which propagates within the subspace (x^1, x^2, x^3). For $\nu = i = 1, 2, 3$, equation (3.10) reads

$$\frac{L^2}{z^2}\,\partial^2_0 A_i - \nabla^2_4 A_i - \frac{1}{z}\,\partial_4 A_i = 0 \tag{3.11}$$

and for $\nu = 0$ equation (3.10) reads

$$\frac{z}{L}\left(\frac{L^2}{z^2}\,\partial^2_0 A_0 - \nabla^2_4 A_0 - \frac{1}{z}\,\partial_4 A_0\right) - \frac{1}{z}\,\partial_0 A_4 = 0 \tag{3.12}$$

Since we seek a solution which exists within the three-dimensional subspace (x^1, x^2, x^3) the constraint

$$A_4 = 0 \tag{3.13}$$

is a reasonable one. When it is applied, equations (3.11) and (3.12) yield

$$\frac{L^2}{z^2}\frac{\partial^2 A_\mu}{\partial t^2} - \frac{\partial^2 A_\mu}{\partial z^2} - \frac{1}{z}\frac{\partial A_\mu}{\partial z} - \nabla^2 A_\mu = 0 \tag{3.14}$$

where $\mu = 0, 1, 2, 3$.

As a solution to equation (3.14) we try the following:

$$A_\mu = A_{\mu 0}\, g(z) \exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{3.15}$$

where the $A_{\mu 0}$ are constants and $\mathbf{x} = (x^1, x^2, x^3)$. Inserting equation (3.15) into equation (3.14), we obtain

$$(pz)^2 \frac{d^2g}{d(pz)^2} + (pz)\frac{dg}{d(pz)} - [(pz)^2 + (iEL)^2]g = 0 \tag{3.16}$$

Once again we recognize the modified Bessel equation. Its solution is

$$g(z) = g(-z) = K_{iEL}(pz) \tag{3.17}$$

Equations (3.15) and (3.17) yield

$$A_\mu = A_{\mu 0} K_{iEL}(p\,|z|)\exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{3.18}$$

Inverting equation (3.4) yields

$$\tilde{A}_\lambda = V^\alpha_\lambda A_\alpha$$

This relation, along with equations (3.9), (3.13), and (3.18), yields

$$\tilde{A}_i = A_{i0} K_{iEL}(p\,|z|)\exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{3.19}$$

$$\tilde{A}_0 = \tilde{A}_4 = 0 \tag{3.20}$$

where $i = 1, 2, 3$, and we have set $A_{00} = 0$. Equations (3.19) and (3.20) represent an ordinary electromagnetic wave propagating in three-dimensional space as far as the (x^1, x^2, x^3, t) dependence is concerned. The Bessel function factor betrays the true four-dimensional character of the wave, but at the same time confines it to a slice of four-dimensional space near $z = 0$. Equations (3.19) and (3.20) may therefore be regarded as an electromagnetic wave which is trapped within the three-dimensional subspace (x^1, x^2, x^3). To an observer who can experience only electromagnetic waves and who lives on a scale large compared to the extent of the electromagnetic wave in the z direction, the four-dimensional space would appear to be three-dimensional.

4. FOUR-DIMENSIONAL FIELD EQUATIONS

We now generalize the results of Section 3 by showing that the field equations for the photon in a five-dimensional Rindler frame can compactify and display the form of the conventional field equations of ordinary four-dimensional Minkowski space-time.

With the five-dimensional source j_ν, included, equations (3.1) become

$$g^{\lambda\mu} D_\lambda D_\mu \tilde{A}_\nu = -4\pi j_\nu \tag{4.1}$$

These then lead to the inhomogeneous analog of equation (3.14),

$$\left(\frac{L^2}{z^2} - 1\right)\frac{\partial^2 A_\mu}{\partial t^2} - \frac{\partial^2 A_\mu}{\partial z^2} - \frac{1}{z}\frac{\partial A_\mu}{\partial z} + \frac{\partial^2 A_\mu}{\partial t^2} - \nabla^2 A_\mu = 4\pi J_\mu \tag{4.2}$$

where $\mu = 0, 1, 2, 3$ and

$$J_i = j_i, \qquad i = 1, 2, 3$$

$$J_0 = \frac{L}{z} j_0$$

In equation (4.2) we have added and subtracted $\partial^2 A_\mu/\partial t^2$ in anticipation of what is to follow.

Expressing A_μ as

$$A_\mu = \int d\omega\, a(\omega, z) b_\mu(\omega, \mathbf{x}) \exp(i\omega t) \tag{4.3}$$

where $\mathbf{x} = (x_1, x_2, x_3)$ and inserting it into equation (4.2) yields

$$-\frac{1}{z^2} \int d\omega \left\{ (\omega z)^2 \frac{\partial^2 a}{\partial(\omega z)^2} + (\omega z) \frac{\partial a}{\partial(\omega z)} \right.$$
$$\left. - [(\omega z)^2 + (i\omega L)^2] a \right\} b_\mu \exp(i\omega t)$$
$$+ \frac{\partial^2 A_\mu}{\partial t^2} - \nabla^2 A_\mu = 4\pi J_\mu \tag{4.4}$$

With $a(\omega, z)$ given by

$$a(\omega, z) = K_{i\omega L}(\omega z) \tag{4.5}$$

the first two lines of equation (4.4) vanish. Upon integrating what is left over z we obtain

$$\frac{\partial^2 A'_\mu}{\partial t^2} - \nabla^2 A'_\mu = 4\pi J'_\mu \tag{4.6}$$

where

$$J'_\mu = \int dz\, J_\mu, \qquad A'_\mu = \int dz\, A_\mu \tag{4.7}$$

are the ordinary four-dimensional photon field and current. Equations (4.3), (4.5), and (4.7) yield

$$A'_\mu = \int dz\, d\omega\, K_{i\omega L}(\omega z) b_\mu(\omega, x) \exp(i\omega t) \tag{4.8}$$

Equations (4.6) and (4.8) are the standard field equations and vector potential of electromagnetism in four-dimensional Minkowski space-time.

The Bessel function factor attenuates as $\exp(-\omega z)$ and so confines A_μ to a region near $z = 0$ in the five-dimensional Rindler frame. If j_ν is the current of a scalar field, then it is similarly confined, as was demonstrated in Section 2. In the next section we will study the spinor field and show that its current is confined also. This then will compactify electromagnetism to four space-time dimensions for the cases when the electromagnetic current is comprised of scalar and spinor fields.

5. THE SPINOR FIELD

In a general coordinate frame in four-dimensional space-time a Dirac spinor ψ obeys the following equation of motion (Weinberg, 1972):

$$\gamma^\alpha V_\alpha{}^\mu \partial_\mu \psi + \tfrac{1}{2}\gamma^\alpha \sigma^{\beta\lambda} V_\beta{}^\nu V_\alpha{}^\mu (D_\mu V_{\lambda\nu})\psi = 0 \tag{5.1}$$

where α, β, and λ are Lorentz indices and μ and ν are coordinate indices. The $V_\alpha{}^\mu$ are inverse vierbein components, the γ^α are 4×4 gamma matrices satisfying

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\eta^{\alpha\beta} \tag{5.2}$$

and $\eta^{\alpha\beta}$ is the Minkowski tensor, diag$[-1, 1, 1, 1]$. The $\sigma^{\beta\lambda}$ are 4×4 matrices satisfying the following commutation relations:

$$[\sigma^{\alpha\beta}, \sigma^{\gamma\delta}] = \eta^{\gamma\beta}\sigma^{\alpha\delta} - \eta^{\gamma\alpha}\sigma^{\beta\delta} + \eta^{\delta\beta}\sigma^{\gamma\alpha} - \eta^{\delta\alpha}\sigma^{\gamma\beta} \tag{5.3}$$

and D_μ is the covariant derivative,

$$D_\mu V_{\lambda\nu} = \partial_\mu V_{\lambda\nu} - \Gamma^\sigma_{\mu\nu} V_{\lambda\sigma} \tag{5.4}$$

We may use all of the above in five-dimensional Rindler space-time if we let all indices run from 0 through 4 and define $\eta^{\alpha\beta}$ as the 5×5 Minkowski tensor, diag$[-1, 1, 1, 1, 1]$. The $V_\alpha{}^\mu$ in five-dimensional Rindler space-time can be obtained by taking the inverse of the expression for $V_\mu{}^\alpha$ given in equation (3.9). This gives

$$V_\alpha{}^\mu = \delta_\alpha{}^\mu + \left(\frac{L}{z} - 1\right)\delta_0{}^\mu \delta_\beta{}^0 \tag{5.5}$$

The γ^α are chosen to satisfy equation (5.2). In what follows we will use the following 4×4 representation for them:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\gamma^k = \begin{pmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{pmatrix}, \qquad k = 1, 2, 3 \tag{5.6}$$

$$\gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

where the σ_k are the Pauli matrices. From equations (5.2) and (5.3) we obtain

$$\sigma^{\alpha\beta} = \tfrac{1}{4}[\gamma^\alpha, \gamma^\beta] \tag{5.7}$$

As we will see, the spinor ψ in five-dimensional Rindler space-time is permitted a mass, even though equation (5.1) contains no mass term.

Equations (3.7), (3.8), (5.4), (5.5), and (5.7) may be used to write equation (5.1) as

$$\frac{L}{z}\gamma^0\partial_0\psi + \frac{1}{2z}\gamma^4\psi + \gamma^k\partial_k\psi + \gamma^4\partial_4\psi = 0 \tag{5.8}$$

where $k = 1, 2, 3$.

We now express ψ as

$$\psi = \begin{pmatrix} \zeta_1(z)\chi \\ \zeta_2(z)\chi \end{pmatrix} \exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{5.9}$$

where χ satisfies $\sigma\cdot\mathbf{p}\chi = p\chi$. Equations (5.6), (5.8), and (5.9) then yield the coupled equations

$$\frac{1}{z}\left(iEL + \frac{1}{2}\right)\zeta_2 - ip\zeta_1 + \frac{d\zeta_2}{dz} = 0 \tag{5.10}$$

$$\frac{1}{z}\left(-iEL + \frac{1}{2}\right)\zeta_1 + ip\zeta_2 + \frac{d\zeta_1}{dz} = 0 \tag{5.11}$$

Two sets of solutions to these equations are readily obtainable.

Solving equation (5.11) for ζ_2 and inserting the result into equation (5.10) yields the modified Bessel equation

$$(pz)^2\frac{d^2\zeta_1}{d(pz)^2} + (pz)\frac{d\zeta_1}{d(pz)} - \left[(pz)^2 + \left(\frac{1}{2} - iEL\right)^2\right]\zeta_1 = 0 \tag{5.12}$$

Equations (5.11) and (5.12) then yield

$$\zeta_1 = K_a(pz) \tag{5.13}$$

$$\zeta_2 = \frac{EL + i/2}{pz}K_a(pz) + i\frac{dK_a(pz)}{d(pz)} \tag{5.14}$$

where $a = 1/2 - iEL$.

Similarly, solving equation (5.10) for ζ_1 and inserting the result into equation (5.11) yields the modified Bessel equation

$$(pz)^2\frac{d^2\zeta_2}{d(pz)^2} + (pz)\frac{d\zeta_2}{d(pz)} - \left[(pz)^2 + \left(\frac{1}{2} + iEL\right)^2\right]\zeta_2 = 0 \tag{5.15}$$

Equations (5.10) and (5.15) then yield

$$\zeta_1' = \frac{EL - i/2}{pz} K_b(pz) - i\frac{dK_b(pz)}{d(pz)} \tag{5.16}$$

$$\zeta_2' = K_b(pz) \tag{5.17}$$

where $b = 1/2 + iEL$. In equations (5.16) and (5.17) we have given ζ_1 and ζ_2 the new names ζ_1' and ζ_2', respectively, for clarity in what is to follow.

The solution to equation (5.1) which we choose to represent the spinor field in the Rindler frame is a linear combination of the above solutions. We write it as

$$\psi = \begin{pmatrix} \phi_1(z)\chi \\ \phi_2(z)\chi \end{pmatrix} \exp(i\mathbf{p}\cdot\mathbf{x} - iEt) \tag{5.18}$$

where

$$\phi_1 = \zeta_1 + \zeta_1', \qquad \phi_2 = \zeta_2 + \zeta_2'$$

An immediate consequence of this choice is

$$\phi_1{}^* = \phi_2 \tag{5.19}$$

By multiplying equation (5.8) on the left by $\bar{\psi} = \psi^\dagger\gamma^0$ and the Hermitian conjugate of equation (5.8) on the right by $\gamma^0\psi$ and subtracting the resulting equations we obtain

$$\partial_0\left(\frac{L}{z}\bar{\psi}\gamma^0\psi\right) + \partial_k(\bar{\psi}\gamma^k\psi) + \frac{1}{z}\bar{\psi}\gamma^4\psi = 0 \tag{5.20}$$

where k is summed from 1 through 4.

With the spinor current J_μ defined as

$$J^0 = \frac{L}{z}\bar{\psi}\gamma^0\psi \tag{5.21}$$

$$J^k = \bar{\psi}\gamma^k\psi, \qquad k = 1, 2, 3, 4 \tag{5.22}$$

we recognize equation (5.20) as the statement that the covariant divergence of the spinor current in the Rindler frame is zero.

Equations (5.18), (5.19), and (5.22) yield

$$J^4 = 0$$

i.e., there is no particle flux in the z direction.

As in the case of the scalar and electromagnetic fields, ψ attenuates exponentially with pz in the asymptotic region. The probability density J^0

associated with ψ therefore also attenuates. This confines ψ to a region of four-dimensional space near $z = 0$ where it masquerades as a field in three-dimensional space. In addition, a solution to equation (5.1) exists for which $E > p$. The field ψ may therefore possess a mass even though equation (5.1) is massless.

6. DISCUSSION

We have found solutions in five-dimensional Rindler space-time M_5 to the field equations of scalar, spinor, and electromagnetic fields. These solutions are trapped in the four-dimensional subspace M_4 which exists near the Rindler horizon at $z = 0$. We may think of these solutions as representing particles which are trapped in a gravity well whose minimum is at $z = 0$. If a particle strays to a value of z above M_4 it is pushed back to M_4 by a gravitational force. In Section 4 we showed that all of electromagnetism can take place in M_4, i.e., currents, charges, and fields obey the laws we are familiar with, laws which are a consequence of the four-dimensional nature of our space-time. Although we have not studied the case of the gauge fields of the electroweak and strong interactions, it seems obvious that the particles associated with these fields should be trapped in the gravity well also and therefore must reside in M_4.

At this point it is natural to ask the following question. Does the universe we live in actually contain four space dimensions, i.e., is it M_5 of the previous paragraph, and do we reside in M_4 near $z = 0$? Any measurement we could make, including those associated with our senses, is consistent with this possibility.

One advantage in choosing M_5 as a model for the universe is that it can provide an elementary particle with mass without the need for a mass term in the corresponding field equation. Mass may therefore be introduced without spoiling gauge invariance. If the Higgs boson remains undiscovered, this alternative route to mass may turn out to be useful.

One may ask, assuming our space-time to be five dimensional, why must it be Rindler in nature? What makes the z axis of our coordinate system different from the other axes? Specifically, why does our universe appear to be accelerating in the z direction? in our next paper we provide an answer to this question and, in doing so, propose a cosmological model which is free of many of the difficulties associated with the Big Bang model. The new model will describe a universe which is immensely old, consistent with general relativity, and which expands, but whose origin is not associated with a Big Bang. It will also offer an observational test of its validity.

We have discussed solutions to the field equations for which $E > p$ (scalar and spinor fields) and solutions for which $E = p$ (electromagnetic field), but what about solutions for which $E < p$? These appear to be valid as well. (Any values of E and p are allowed.) These solutions represent tachyons, and at present we see no way of avoiding them.

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